

Vacuum neutrino oscillations of solar neutrinos and lepton mass matrices

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We consider the case that the solar neutrino deficit is due to vacuum oscillations. The lepton mass matrices with nearly bimaximal mixings are needed in order to explain both the solar and atmospheric neutrino deficit. A texture with the symmetry of flavor democracy or S_3 has been investigated by taking account of the symmetry breaking terms of the charged lepton mass matrix. It is found that predicted mixings can be considerably changed from the neutrino mixings $\sin^2 2\theta_\odot \approx 1$ and $\sin^2 2\theta_{\text{atm}} \approx 8/9$ at the symmetric limit. The correlation between $|U_{e3}|$ and $|U_{e1}U_{e2}^*|$ is also presented. The test of the model is discussed by focusing on the three flavor analyses in the solar neutrinos, atmospheric neutrinos, and long baseline experiments. [S0556-2821(99)07501-3]

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Neutrino flavor oscillations provide information of the fundamental properties of neutrinos such as mass and flavor mixing. In recent years, there is growing experimental evidence of neutrino oscillations. One exciting example is the atmospheric neutrino deficit [1–3] as well as the solar neutrino deficit [4]. Super-Kamiokande [5] presented the near-maximal neutrino flavor oscillation in atmospheric neutrinos. Recent Super-Kamiokande data also suggest that vacuum oscillation regions are favored in the analysis of day-night spectra and energy shape of the solar neutrino [6–8]. Therefore, three flavor analyses are very important for the vacuum oscillation of solar neutrinos [9,10]. Results of those analyses can give constraints on the structure of the lepton mass matrices. Therefore, it is urgent to make clear predictions quantitatively in the lepton mass matrix models which give near-maximal solar and atmospheric vacuum oscillations [11–13].

There is a typical texture of the lepton mass matrix with nearly bimaximal mixing of three neutrinos, which is derived from the symmetry of the lepton flavor democracy by Fritzsch and Xing [14], or from the $S_{3L} \times S_{3R}$ symmetry of the left-handed Majorana neutrino mass matrix given by Fukugita, Tanimoto, and Yanagida [15]. These models can give identical predictions for the neutrino mixings $\sin^2 2\theta_\odot \approx 1$ and $\sin^2 2\theta_{\text{atm}} \approx 8/9$ although those depend on the symmetry breaking patterns. However, these predictions are considerably changed if the symmetry breaking terms are correctly taken into account. In this paper, we investigate the quantitative predictions on the neutrino flavor mixings including symmetry breaking terms, which become significant for long baseline (LBL) neutrino oscillation experiments. In particular, LBL experiments of $\nu_\mu \rightarrow \nu_e$ can test the prediction of U_{e3} in the model. The first LBL reactor experiment CHOOZ has already reported a bound of the neutrino oscillation [16], which gives a strong constraint of the flavor mixing pattern. The LBL accelerator experiment K2K [17] is planned to begin taking data in the next year, whereas the MINOS [18] and ICARUS [19] experiments will start in the first year of the next century. Thus the lepton mass matrix model will be tested in the near future.

Our conservative approach is to assume that oscillations need only account for the solar and atmospheric neutrino data. Since the result of LSND [20] awaits confirmation by KARMEN experiment [21], we do not take into consideration the LSND data in this paper. Recent results of atmospheric neutrinos at Super-Kamiokande [5] suggest $\nu_\mu \rightarrow \nu_\tau$ oscillation with the near-maximal mixing. Since the CHOOZ result [16] excludes the large neutrino oscillation of $\nu_\mu \rightarrow \nu_e$ as far as $\Delta m^2 \geq 9 \times 10^{-4} \text{ eV}^2$, the large mixing between ν_μ and ν_τ is a reasonable interpretation for the atmospheric ν_μ deficit. Our starting point as to neutrino mixings is the near-maximal $\nu_\mu \rightarrow \nu_\tau$ oscillation with

$$\Delta m_{\text{atm}}^2 = 10^{-3} - 10^{-2} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} \geq 0.8, \quad (1)$$

which constrain neutrino oscillations in LBL experiments. On the other hand, recent solar neutrino data of Super-Kamiokande [6] favors vacuum long-wavelength oscillations with $\Delta m_\odot^2 \approx 10^{-10} \text{ eV}^2$ and the near-maximal mixing in the analysis of day-night spectra and energy shape.

The texture of the lepton mass matrices with nearly bimaximal mixing of three neutrinos was presented based on the democratic mass matrix as follows [14,15]: the charged lepton mass matrix is

$$M_l = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \delta_l & 0 & 0 \\ 0 & \rho_l & 0 \\ 0 & 0 & \epsilon_l \end{pmatrix}, \quad (2)$$

where the second matrix is the symmetry breaking terms, which were given for quark mass matrices by Koide [22], and δ_l, ρ_l and ϵ_l are complex parameters in general. By neglecting CP violating phases and using $c_l \gg \epsilon_l \gg \rho_l, \delta_l$, this matrix is diagonalized approximately as

$$V_l^\dagger M_l V_l = \text{diag}(m_{l1}, m_{l2}, m_{l3}), \quad (3)$$

where

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$$\begin{aligned}
m_{11} &= (\delta_l + \rho_l + \epsilon_l)/3 - \xi_l/6, \\
m_{12} &= (\delta_l + \rho_l + \epsilon_l)/3 + \xi_l/6, \\
m_{13} &= c_l + (\delta_l + \rho_l + \epsilon_l)/3,
\end{aligned} \tag{4}$$

with

$$\xi_l = [(2\epsilon_l - \rho_l - \delta_l)^2 + 3(\rho_l - \epsilon_l)^2]^{1/2}. \tag{5}$$

The unitary matrix V_l is given as $V_l = FL$, where

$$\begin{aligned}
F &= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}, \\
L &\simeq \begin{pmatrix} \cos \theta_l & -\sin \theta_l & \lambda_l \sin 2\theta_l \\ \sin \theta_l & \cos \theta_l & \lambda_l \cos 2\theta_l \\ -\lambda_l \sin 3\theta_l & -\lambda_l \cos 3\theta_l & 1 \end{pmatrix},
\end{aligned} \tag{6}$$

with

$$\tan 2\theta_l \simeq -\sqrt{3} \frac{\rho_l - \delta_l}{2\epsilon_l - \rho_l - \delta_l}, \quad \lambda_l = \frac{1}{\sqrt{2}} \frac{1}{3c_l} \xi_l. \tag{7}$$

If a special condition $\delta_l = -\rho_l$ is taken, one can obtain the familiar relation $|L_{21}| \simeq \sin \theta_l \simeq \sqrt{m_{11}/m_{12}}$, which was used in Refs. [14] and [15]. However, this condition is not guaranteed in the framework of the model. In our following analyses, a relation between δ_l and ρ_l is given only by the mass $m_{11} = m_e$ as seen in Eq. (4), and so the value of L_{21} is given arbitrary. The L_{13} and L_{31} mixings are suppressed compared with L_{12} and L_{21} because λ_l is $\mathcal{O}(m_\mu/m_\tau)$. On the other hand, the L_{23} and L_{32} mixings are almost fixed except for phases such as

$$|L_{23}| \simeq |L_{32}| \simeq \frac{1}{\sqrt{2}} \frac{m_\mu}{m_\tau} \simeq 0.04. \tag{8}$$

Let us turn to the neutrino sector. Assuming that the neutrinos are of the Majorana type, the neutrino mass matrix is

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \delta_\nu & 0 & 0 \\ 0 & \rho_\nu & 0 \\ 0 & 0 & \epsilon_\nu \end{pmatrix}, \tag{9}$$

where the first matrix is the S_{3L} invariant one and the second is the symmetry breaking one. It is noted that $c_\nu = 0$ and $c_\nu = \mathcal{O}(1 \text{ eV})$ were taken in Ref. [14] and in Ref. [15], respectively. Since the mass matrix of the neutrino is still diagonal in both models, the same numerical results are obtained for flavor mixings. The neutrino masses are determined being independent of flavor mixings in this model. So the parameters δ_ν, ρ_ν and ϵ_ν are easily constrained by putting $\Delta m_{21}^2 = \Delta m_{\odot}^2$ and $\Delta m_{31}^2 = \Delta m_{\text{atm}}^2$.

The neutrino mixing matrix $U_{\alpha i}$ is determined by only V_l of the charged leptons as follows:

$$U = V_l^\dagger = L^\dagger F^\dagger. \tag{10}$$

Then, the relevant mixing parameters of solar neutrinos are

$$\begin{aligned}
U_{e1} &= \frac{1}{\sqrt{2}} L_{11}^* + \frac{1}{\sqrt{6}} L_{21}^*, \\
U_{e2} &= -\frac{1}{\sqrt{2}} L_{11}^* + \frac{1}{\sqrt{6}} L_{21}^*, \\
U_{e3} &= -\frac{2}{\sqrt{6}} L_{21}^*,
\end{aligned} \tag{11}$$

where $|L_{11}^*|^2 \simeq 1 - |L_{21}|^2$ and L_{31} is neglected due to the suppression factor λ_l . These mixings are determined only by L_{21} , which is still an unknown parameter because there remains at least one undetermined parameter in the mass matrix after fixing the charged lepton masses m_e , m_μ and m_τ , as seen in Eq. (4). If the complex phases are allowed in the symmetry breaking terms, there are more unknown parameters. On the other hand, the relevant mixing parameters of atmospheric neutrinos are

$$U_{\mu 3} = -\frac{2}{\sqrt{6}} L_{22}^* + \frac{1}{\sqrt{3}} L_{32}^*, \quad U_{\tau 3} = -\frac{2}{\sqrt{6}} L_{23}^* + \frac{1}{\sqrt{3}} L_{33}^*, \tag{12}$$

where $|L_{22}^*|^2 \simeq 1 - |L_{21}|^2 - |L_{23}|^2$ and $|L_{33}^*|^2 \simeq 1 - |L_{32}|^2$. Thus these mixings are determined by L_{21} and L_{32} .

As seen in Eq. (11), there is a correlation between $4|U_{e1}U_{e2}^*|^2$ and $|U_{e3}|$; both are relevant quantities for the oscillation probability of solar neutrinos as follows:

$$\begin{aligned}
P(\nu_e \rightarrow \nu_e) &\simeq 1 - 4|U_{e1}U_{e2}^*|^2 \sin^2 \frac{\Delta m_{21}^2 L}{4E} \\
&\quad - 2|U_{e3}|^2 (1 - |U_{e3}|^2).
\end{aligned} \tag{13}$$

Since those are given in terms of $L_{21} = |L_{21}| \exp(ip)$, we can present an allowed region on the $(|U_{e3}|, 4|U_{e1}U_{e2}^*|^2)$ plane by changing $|L_{21}|$ and the phase p . In Fig. 1, the allowed region is shown between two curves for the case of $p = 0^\circ$ and $p = 90^\circ$. If $\Delta m_{31}^2 \geq 9 \times 10^{-4} \text{ eV}^2$, the mixing $|U_{e3}|$ is constrained by the CHOOZ experiment [16], in which the oscillation probability is expressed as

$$\begin{aligned}
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) &\simeq 1 - 4|U_{e3}|^2 (1 - |U_{e3}|^2) \\
&\quad \times \sin^2 \frac{\Delta m_{31}^2 L}{4E}.
\end{aligned} \tag{14}$$

For example, the CHOOZ result gave constraints $\sin^2 2\theta_{\text{CHOOZ}} \leq \{0.12, 0.20, 0.75\}$ for $\Delta m_{31}^2 \simeq \{5, 2, 1\} \times 10^{-3} \text{ eV}^2$, which correspond to $|U_{e3}| \leq \{0.18, 0.23, 0.50\}$, respectively. Thus the constraint of $|U_{e3}|$ is strongly dependent on

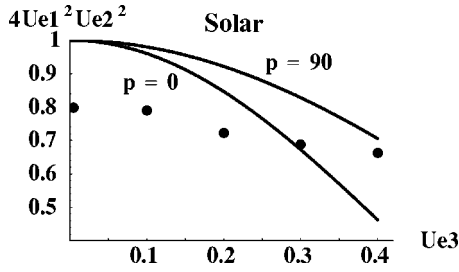


FIG. 1. Allowed region on the $(|U_{e3}|, 4|U_{e1}U_{e2}^*|^2)$ plane, which is the one between two curves for $p=0^\circ$ and $p=90^\circ$. Black points denote the best fits of solar neutrinos in the three flavor analyses in Ref. [10] for each $|U_{e3}|$. For $|U_{e3}|=0.0, 0.2$, and 0.4 , the values of χ^2_{\min} are 3.6, 4.2, and 5.6, respectively. The allowed region at 95% C.L. completely covers our predicted one.

Δm_{31}^2 . This fact is advantageous for testing the model because the solar neutrino data determines the best fit of $(|U_{e3}|, 4|U_{e1}U_{e2}^*|^2)$ being independent of Δm_{31}^2 . We show the best-fit points in the three flavor analyses in Ref. [10] by four black points for each $|U_{e3}|$. If these best-fit points are reliable, we can find in Fig. 1 that $|U_{e3}| \approx 0.3-0.4$ is favored in the present model, which means $\Delta m_{31}^2 \leq 10^{-3} \text{ eV}^2$ in the CHOOZ data. LBL experiments can test this prediction in the future.

However, it is important to comment on the allowed region at 95% C.L. in Ref. [10], which completely covers our predicted one. Thus the prediction of $|U_{e3}|$ in the model is less predictive at the present. Moreover, for $|U_{e3}|=0.0, 0.2$, and 0.4 , the values of χ^2_{\min} are 3.6, 4.2, and 5.6, respectively. This means that smaller values of $|U_{e3}|$ are preferred. Therefore, we need three flavor analyses of the solar neutrino oscillation including new data of the energy spectrum in Super-Kamiokande [6,7].

LBL experiments provide an important test of the model because $|U_{e3}|$ is a key ingredient for the $\nu_\mu \rightarrow \nu_e$ oscillation as follows:

$$P(\nu_\mu \rightarrow \nu_e) \approx 4|U_{e3}U_{\mu 3}^*|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} \approx \frac{8}{3}|U_{e3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}, \quad (15)$$

where $|U_{\mu 3}| = 2/\sqrt{6}$ and the small CP violating term is neglected. Then, the expected maximal oscillation probability

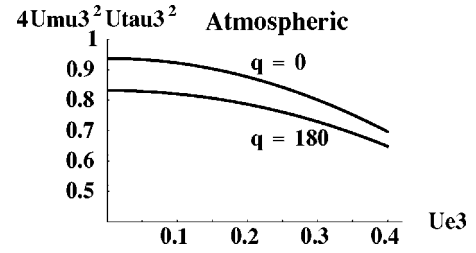


FIG. 2. Allowed region on the $(|U_{e3}|, 4|U_{\mu 3}U_{\tau 3}^*|^2)$ plane which is the one between two curves for $q=0^\circ$ and $q=180^\circ$.

is 0.06 for $\Delta m_{31}^2 \approx 5 \times 10^{-3} \text{ eV}^2$ at K2K [23] due to the constraint of the CHOOZ result [16].

Let us consider the case of the atmospheric neutrino. By using $L_{32} \approx 0.04 \exp(iq)$, we can estimate $4|U_{\mu 3}U_{\tau 3}^*|^2$ as a function of $|U_{e3}|$, where $q=0^\circ-180^\circ$ and L_{23} is determined by unitarity. We present an allowed region on the $(|U_{e3}|, 4|U_{\mu 3}U_{\tau 3}^*|^2)$ plane in Fig. 2, where the allowed region is shown between two curves for the case of the phase $q=0^\circ$ and $q=180^\circ$. It is found that $4|U_{\mu 3}U_{\tau 3}^*|^2$ could deviate significantly from 8/9, which was presented in Refs. [14,15], due to the symmetry breaking terms in the model. The LBL experiments of $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_e$ enable us to test our predictions in the future.

The summary is given as follows. The mass matrix model based on the democratic form has been investigated focusing on the recent data of solar neutrinos and atmospheric neutrinos. If the solar neutrino deficit is due to the vacuum oscillation as suggested by recent Super-Kamiokande data, nearly bimaximal mixings are needed in order to explain both the solar and atmospheric neutrino deficit. In the present model, those large mixings are derived from the charged lepton sector while the neutrino mass matrix is a diagonal one. It is remarked that the symmetry breaking term of the charged lepton mass matrix is very important for predicting mixing angles. The model will be tested by the precise determination of the mixings and Δm^2 in the solar neutrinos, atmospheric neutrinos and LBL experiments in the near future.

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- [1] K. S. Hirata *et al.*, Phys. Lett. B **205**, 416 (1988); **280**, 146 (1992); D. Casper *et al.*, Phys. Rev. Lett. **66**, 2561 (1991); R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992).
- [2] NUSEX Collaboration, M. Aglietta *et al.*, Europhys. Lett. **15**, 559 (1991); SOUDAN2 Collaboration, W. W. M. Allison *et al.*, Phys. Lett. B **391**, 491 (1997); Fréjus Collaboration, K. Daum, Z. Phys. C **66**, 417 (1995); MACRO Collaboration, S. Ahlen *et al.*, Phys. Lett. B **357**, 481 (1995).
- [3] Y. Fukuda *et al.*, Phys. Lett. B **335**, 237 (1994).
- [4] GALLEX Collaboration, P. Anselmann *et al.*, Phys. Lett. B

- 327**, 377 (1994); W. Hampel *et al.*, *ibid.* **388**, 384 (1996); SAGE Collaboration, J. N. Abdurashitov *et al.*, *ibid.* **328**, 234 (1994); Homestake Collaboration, Nucl. Phys. B (Proc. Suppl.) **38**, 47 (1995); Kamiokande Collaboration, *ibid.* **38**, 55 (1995); Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1998).
- [5] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
- [6] Y. Suzuki, invited talk at Neutrino 98, Takayama, Japan, 1998.

- [7] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, *Phys. Rev. D* **58**, 096016 (1998).
- [8] A. J. Baltz, A. S. Goldhaber, and M. Goldhaber, hep-ph/9806540.
- [9] S. P. Mikheyev and A. Yu. Smirnov, *Phys. Lett. B* **429**, 343 (1998); B. Faïd, G. L. Fogli, E. Lisi, and M. Montanino, BARI-TH/299-98, hep-ph/9805293.
- [10] P. Oslund and G. Vigdel, CERN-TH/98-200, hep-ph/9806339.
- [11] V. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, MADPH-98-1063, hep-ph/9806387, v3.
- [12] M. Jezabek and Y. Sumino, hep-ph/9807310.
- [13] Y. Nomura and T. Yanagida, *Phys. Rev. D* (to be published), hep-ph/9807325.
- [14] H. Fritzsch and Z. Xing, *Phys. Lett. B* **372**, 265 (1996); hep-ph/9807234; Z. Xing, DPNU-98-12, hep-ph/9804433.
- [15] M. Fukugita, M. Tanimoto, and T. Yanagida, *Phys. Rev. D* **57**, 4429 (1998).
- [16] CHOOZ Collaboration, M. Apollonio *et al.*, *Phys. Lett. B* **420**, 397 (1998).
- [17] Y. Suzuki, in *Proceedings of Neutrino 96*, Helsinki, 1996, edited by K. Enqvist *et al.* (World Scientific, Singapore, 1997), p. 237.
- [18] S. G. Wojcicki, in *Proceedings of Neutrino 96* (Ref. [17]), p. 231.
- [19] ICARUS Collaboration, P. Cennini *et al.*, LNGS-94/99-I, 1994; F. Pietropaola, invited talk at Neutrino 98, Takayama, Japan, 1998.
- [20] LSND Collaboration, C. Athanassopoulos *et al.*, *Phys. Rev. Lett.* **75**, 2650 (1995); **77**, 3082 (1996); *Phys. Rev. C* **54**, 2685 (1996); nucl-ex/9706006 (1997); J. E. Hill, *Phys. Rev. Lett.* **75**, 2654 (1995).
- [21] KARMEN Collaboration, *Nucl. Phys. B (Proc. Suppl.)* **38**, 235 (1995); B. Zeitnitz, invited talk at Neutrino 98, Takayama, Japan, 1998.
- [22] Y. Koide, *Phys. Rev. D* **28**, 252 (1983); **39**, 1391 (1989).
- [23] M. Tanimoto, *Phys. Lett. B* **435**, 373 (1998).